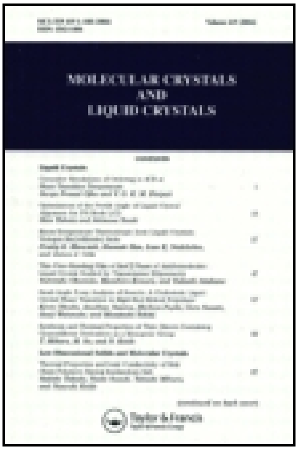

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Periodic Disturbances in Cylindrically Layered Smectic A

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The hydrodynamic equations for a cylindrically layered sample of smectic A liquid crystal are considered. Governing equations for the flow, pressure and smectic layer undulations are derived using a perturbation ansatz. Expressions for the radial flow velocity and the layer displacement are derived analytically and presented in the purely radial flow case. An expression for the relaxation rate is found and a bound for the spatial wave number is determined when zenithal flow is also considered. Plots of the solutions are given for illustration. Vindication of previous results is discussed.

Keywords: cylindrical layers; dynamics of liquid crystals; smectic A liquid crystals; stability

1. INTRODUCTION

We consider the hydrodynamic equations of a cylindrically layered smectic A liquid crystal sample and aim to see how the sample behaves under the influence of a sinusoidal perturbation. The problem considered has been motivated by an emergence of recent experimental evidence for the occurrence of stable and unstable domains of cylindrically arranged smectic cylinders [1,2]. It has been suggested that stable or unstable layer undulations may be indicative of the occurrence or non-occurrence of cylindrical domains, respectively, as the layers are a key feature of smectic liquid crystals. Kléman and Parodi [3] initially investigated the subject of cylindrically layered SmA and our intention is to present a rigorous application of theory which will

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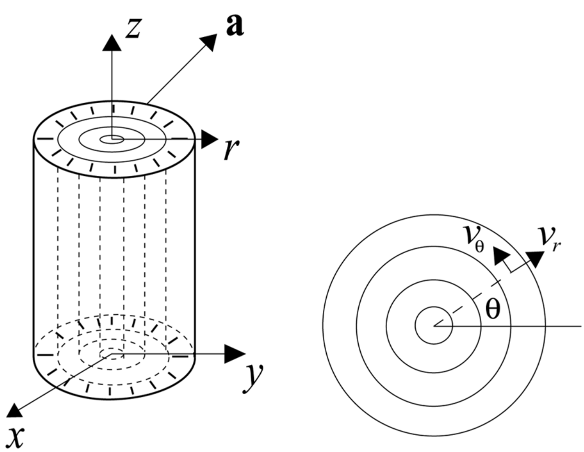
vindicate and elucidate the more speculative results anticipated by an elementary energy analysis in [3]. Diez and Atkinson [4] give relevant impetus to this matter when they describe flow near a corner between two plates, albeit with a nematic liquid crystal. Work here was also aided by the results of Payr, Vanaparthy and Meiburg [5] and Vanaparthy, Meiburg and Wilhelm [6] in which much insight was given to the thought of dynamics in cylindrical domains of a Newtonian fluid.

Using curvilinear coordinate analysis, and the recent dynamic theory for SmA by Stewart [7] (motivated by the theories introduced by Auernhammer *et al.* [8], Payr *et al.* [5], Soddemann *et al.* [9] and Weinan [10]), we shall present a set of dynamic equations for a cylindrical sample of SmA liquid crystal, figuratively shown in Figure 1. We shall introduce general ansatzes for the velocity in the radial direction v_r, the azimuthal velocity v_θ, the zenithal velocity v_z, the pressure p and the layer undulation displacement u. Using the dynamic equations we shall find explicit expressions for these ansatzes in the radial velocity only case. When the azimuthal velocity is also considered, we find an expression for the relaxation rate and a lower bound for the spatial wave number in the z-direction. Plots of the solutions will also be given for illustration.



(a) (b)

FIGURE 1 (a) A representation of the cylindrical layers where **a** is the layer normal, r the radial direction and the bold lines representing the alignment of the director **n**. (b) A top view of the cylinders showing the directions of the radial and azimuthal velocities, v_r and v_θ respectively, and the azimuthal angle θ. The zenithal velocity will be in the direction perpendicular to the page.

2. DYNAMIC THEORY FOR SMA

The dynamic equations proposed by Stewart [7] allow the possibility of the layer normal **a** and the director **n** not necessarily coinciding in non-equilibrium situations. Although we wish to look at this occurrence in the cylindrically layered sample, we assume $\mathbf{a} \equiv \mathbf{n}$ in this work. Noting this, the dynamic equations are as follows.

Describing the compressible smectic layers by $\Phi = r + u(r, \theta, z, t)$, where $u(r, \theta, z, t)$ denotes the perceived undulations of the smectic layers, we may define the layer normal **a** by

$$a_i = \frac{\Phi_{,i}}{|\nabla\Phi|}, \quad a_i a_i = 1. \tag{1}$$

Incompressibility leads to the usual constraint

$$v_{i,i} = 0 \tag{2}$$

where **v** is the velocity. The equations that arise from the balance law for linear momentum are given by

$$\rho\dot{v}_i = \rho F_i - \tilde{p}_{,i} + |\nabla\Phi| a_i J_{j,j} + \tilde{t}_{ij,j}, \tag{3}$$

where ρ is the density (which we assume constant in this investigation), a superimposed dot represents the usual material time derivative, F_i is the external body force per unit mass, $\tilde{p} = p + w_A$ where p is the pressure and w_A is the energy density. We note here that no generalised external body moments to the liquid crystal sample will be supposed. We shall make use of the classical nonlinear energy density [8]

$$w_A = \frac{1}{2}K_1(\nabla \cdot \mathbf{a})^2 + \frac{1}{2}B_0(|\nabla\Phi| - 1)^2. \tag{4}$$

Here, K_1 is a positive elastic constant and B_0 the layer compression constant. **J** is a phase flux term defined by

$$J_i = -\frac{\partial\omega_A}{\partial\Phi_{,i}} + \frac{1}{|\nabla\Phi|}\left[\left(\frac{\partial\omega_A}{\partial a_{p,k}}\right)_{,k} - \frac{\partial\omega_A}{\partial a_p}\right](\delta_{pi} - a_p a_i), \tag{5}$$

which is a natural extension to that introduced by Weinan [8]. The permeation equation is given by

$$\dot{\Phi} = -\lambda_p J_{i,i}, \tag{6}$$

where $\lambda_p \geq 0$ is the permeation coefficient. For convenience, at this early stage of the investigation, we shall only include the viscosity coefficient which is related to the Newtonian fluid viscosity. The viscous stress tensor in this simplified form is therefore $\tilde{t}_{ij} = \alpha_4 A_{ij}$, where $\alpha_4 > 0$ is the viscosity coefficient and $\mathbf{A}$ is the usual rate of strain tensor.

3. THE DYNAMIC EQUATIONS

Motivated by Payr *et al*. [5] and Vanaparthy *et al*. [6], we assume perturbations to the equilibrium state (where $\mathbf{a} = \hat{\mathbf{r}}, \mathbf{v} = \mathbf{0}$, and $u = 0$ for the geometrical set-up shown in Fig. 1) of the form

$$\begin{pmatrix} v_r \\ v_\theta \\ v_z \\ p \\ u \end{pmatrix}(r,\theta,z,t) = \begin{pmatrix} h_r R f_r(r,z) \\ h_\theta \Theta f_\theta(r,z) \\ h_z Z f_z(r,z) \\ P f_p(r,z) \\ U f_u(r,z) \end{pmatrix} e^{-\omega t + i q_\theta \theta}, \tag{7}$$

where h_r, h_θ, and h_z are the usual scale factors, R, Θ, Z, P, and U are arbitrary constants and all of the functions in r and z are as yet undetermined. The exponential part of our ansatz is a radial harmonic undulation in θ, with wavenumber q_θ, and a decay or growth in time with some as yet unknown decay or growth rate ω, to be determined. Inserting (7) into the five dynamic equations contained in equations (2), (3) and (6) we obtain the following governing equations:

$$Rf_r + rR\left(\frac{df_r}{dr}\right) + f_\theta \Theta q_\theta r i + rZ\left(\frac{df_z}{dz}\right) = 0, \tag{8}$$

$$\begin{aligned} &2r^2\alpha_4 R f_r - q_\theta \alpha_4 r^4 \Theta\left(\frac{df_\theta}{dr}\right) i - 2\rho R \omega f_r r^4 - \alpha_4 r^4 R\left(\frac{d^2 f_r}{dz^2}\right) \\ &- \alpha_4 r^4 Z\left(\frac{d^2 f_z}{drdz}\right) + 2r^3 B_0 U\left(\frac{df_u}{dr}\right) - 2r^3\alpha_4 R\left(\frac{df_r}{dr}\right) \\ &+ 2r^4 B_0 U\left(\frac{d^2 f_u}{dr^2}\right) - 2r^4\alpha_4 R\left(\frac{d^2 f_r}{dr^2}\right) + 4KU\left(\frac{d^2 f_u}{dz^2}\right) r^2 q_\theta^2 \\ &- 2KU\left(\frac{d^4 f_u}{dz^4}\right) r^4 - 2KU\left(\frac{d^2 f_u}{dz^2}\right) r^2 - 2KU f_u q_\theta^4 + 2KU f_u q_\theta^2 \\ &+ a_4 r^2 R f_r q_\theta^2 + 2ir^3 a_4 f_\theta \Theta q_\theta + 2r^4 P\left(\frac{df_p}{dr}\right) = 0, \end{aligned} \tag{9}$$

$$2\rho r^5 f_\theta \Theta\omega + 3i\alpha_4 r^2 R f_r q_\theta + 3\alpha_4 r^4 \Theta\left(\frac{df_\theta}{dr}\right) - 2iKUf_u q_\theta^3 + 2q_\theta i r^3 P f_p + 2iKU\left(\frac{d^3 f_u}{dz^2}\right) q_\theta r^2 + 2iKUf_u q_\theta - 2r^3\alpha_4\left(\frac{d^2 f_\theta}{dz^2}\right)\Theta + \alpha_4 r^5\left(\frac{d^2 f_\theta}{dz^2}\right)\Theta + \alpha_4 r^3 Z\left(\frac{df_z}{dz}\right) q_\theta i + \alpha_4 r^3 R\left(\frac{df_r}{dr}\right) q_\theta i + \alpha_4 r^5 \Theta\left(\frac{d^2 f_\theta}{dr^2}\right) = 0, \quad (10)$$

$$2\rho Z f_z w r^2 + \mu_4 r R\left(\frac{df_r}{dz}\right) + \alpha_4 r Z\left(\frac{df_z}{dr}\right) + \alpha_4 r^2 R\left(\frac{d^2 f_r}{drdz}\right) + \alpha_4 r^2 Z\left(\frac{d^2 f_z}{dr^2}\right) + \alpha_4 q_\theta r^2\left(\frac{df_\theta}{dz}\right)\Theta i - \alpha_4 q_\theta^2 Z f_z - 2P\left(\frac{df_p}{dz}\right) r^2 + 2\alpha_4 Z\left(\frac{d^2 f_z}{dz^2}\right) r^2 = 0, \quad (11)$$

$$-Uf_u\omega r^4 + Rf_r r^4 - \lambda_p U B_0\left(\frac{df_u}{dr}\right) r^3 - 2\lambda_p U r^2 K\left(\frac{d^2 f_u}{dz^2}\right) q_\theta^2 + \lambda_p U r^4 K\left(\frac{d^4 f_u}{dz^4}\right) + \lambda_p U r^2 K\left(\frac{d^2 f_u}{dz^2}\right) - \lambda_p U K f_u q_\theta^2 - \lambda_p U r^4 B_0\left(\frac{d^2 f_u}{dr^2}\right) - \lambda_p U K f_u q_\theta^4 = 0, \quad (12)$$

where all the unknown functions f_i are dependent on r and z.

3.1. Radial Perturbations

In the first instance, we shall only deal with functions in the radial direction that depend on r and time t. Setting the two dynamic equations for the azimuthal and zenithal velocities to be negligible, and neglecting q_θ and derivatives in z, we reduce the five dynamic equations to three: the divergence of the velocity vector, the linear momentum in the r direction and the permeation equation, given respectively by

$$f_r(r) + r\left(\frac{d}{dr} f_r(r)\right) = 0, \quad (13)$$

$$\frac{2}{r^2}\alpha_4 R f_r(r) + 2P\left(\frac{d}{dr} f_p(\tau)\right) - 2\rho R f_r(r)\omega + \frac{2}{r} B_0 U\left(\frac{d}{dr} f_u(r)\right) - \frac{2}{r}\alpha_4 R\left(\frac{d}{dr} f_r(r)\right) + 2B_0 U\left(\frac{d^2}{dr^2} f_u(r)\right) - 2\alpha_4 R\left(\frac{d^2}{dr^2} f_r(r)\right) = 0, \quad (14)$$

$$Uf_u(r)\omega r^4 - Rf_r(r)r^4 + \lambda UB_0\left(\frac{d}{dr}f_u(r)\right)r^3 + \lambda Ur^4B_0\left(\frac{d^2}{dr^2}f_u(r)\right) = 0 \tag{15}$$

These are three equations in the three unknown functions f_r, f_u, and f_p. The divergence equation (13) can be solved easily to give

$$f_r(r) = \frac{r_0}{r}, \tag{16}$$

where r_0 is some minimum allowed radius. We note from (7) and (16) that as $r \to \infty$, $u_r \to 0$ and when $r = r_0$, $u_r = \mathrm{Re}^{-\omega \mathrm{t}}$. Therefore at $t = 0$, a constant velocity in the r direction of some arbitrary magnitude R, $R \neq 0$, is supposed. Substituting (16) into (15) leads to a differential equation which we can solve for $f_u(r)$, given by

$$\begin{aligned} f_u(r) = {} & \mathcal{C}J_0(\sqrt{\gamma}r) + \frac{1}{2\lambda_p UB_0\sqrt{\gamma}}Rr_0\pi H_0(\sqrt{\gamma}r) \\ & - \frac{1}{2Y_0(\sqrt{\gamma}r_0)\lambda_p UB_0\sqrt{\gamma}}\boldsymbol{Y}_0(\sqrt{\gamma}r)\,(2C\lambda_p UB_0\sqrt{\gamma}J_0(\sqrt{\gamma}r_0) \\ & + Rr_0\pi H_0(\sqrt{\gamma}r_0) - 2\sqrt{\gamma}\lambda_p UB_0), \end{aligned} \tag{17}$$

where $\mathcal{C}$ is an arbitrary constant, $J_0(x)$ and $Y_0(x)$ are the usual Bessel functions of the first and second kind and $H_0(x)$ is the Struve function (see Abramowitz & Stegun [11] for more details). As before, we note that as $r \to \infty$, $u \to 0$ and when $r = r_0$, $u = U\mathrm{e}^{-\omega \mathrm{t}}$. Again we remark that when $t = 0$ the displacement is an arbitrary non-zero magnitude U. In the same manner as before we may calculate a form for $f_p(r)$ analytically, which we neglect to print for brevity; see [10] for full details.

In general, it is not clear as to how $f_p(r)$ behaves as $r \to \infty$ without plotting it numerically for physical parameters inserted and an estimate for the unknown magnitude C. To find out, we look at the linear momentum equation in the radial direction (14). We can see by inspection that

$$f_r(r), \frac{d}{dr}f_r(r), \frac{d^2}{dr^2}f_r(r), \frac{d}{dr}f_u(r), \frac{d^2}{dr^2}f_u(r) \to 0 \quad \text{as } r \to \infty, \tag{18}$$

as can be deduced from the solutions (16) and (17) and their corresponding derivatives. As physical considerations require the pressure to tend to a constant as $r \to \infty$ we therefore require $df_p/dr \to 0$ as $r \to \infty$. We can see from equations (14) and (18) that this occurs without any restriction placed on the parameter ω. The solutions are therefore bounded. However, this method does not provide a mechanism for the identification of the decay or growth rate ω and so we are led to look at how the system behaves when a additional zenithal velocity is appended. This will resolve the situation, as will be seen below.

3.2. Perturbations in *r* and *z*

In this section, we assume that the azimuthal flow in the θ direction is still negligible, yet include the possibility of an undulating flow in the z-direction, a possibility considered in [5,6] for Newtonian fluids. Following on from the harmonic ansatz used by Kléman and Parodi [1], we expand and now assume an ansatz of the form

$$\begin{pmatrix} u_r \\ u_z \\ p \\ u \end{pmatrix}(r,z,t) = \begin{pmatrix} h_r R f_r(r) \\ h_z Z f_z(r) \\ P f_p(r) \\ U f_u(r) \end{pmatrix} e^{-\omega t + iqz}, \tag{19}$$

with q being the wave number of the oscillations in the z-direction. We state here that we require $q \in \mathbb{R}$ for stable spatial undulations. With this ansatz the four dynamic equations, arising from the divergence of the velocity vector, the linear momentum in the radial and zenithal directions and the permeation equation, become, respectively

$$Rf_r + rR\left(\frac{df_r}{dr}\right) + irZqf_z = 0, \tag{20}$$

$$\begin{aligned} &2r^2\alpha_4 Rf_r - 2\rho R\omega f_r r^4 + q^2\alpha_4 r^4 Rf_r - iq\alpha_4 r^4 Z\left(\frac{df_z}{dr}\right) + 2r^3 B_0 U\left(\frac{df_u}{dr}\right) \\ &\quad - 2r^3\alpha_4 R\left(\frac{df_r}{dr}\right) + 2r^4 B_0 U\left(\frac{d^2 f_u}{dr^2}\right) - 2r^4\alpha_4 R\left(\frac{d^2 f_r}{dr^2}\right) \\ &\quad - 2q^4 KU f_u r^4 + 2q^2 KU f_u r^2 + 2ir^4 P\left(\frac{df_p}{dr}\right) = 0, \end{aligned} \tag{21}$$

$$\begin{aligned} &2\rho Z f_z \omega r^2 + iq\alpha_4 r R f_r + \alpha_4 r Z\left(\frac{df_z}{dr}\right) + iq\alpha_4 r^2 R\left(\frac{df_r}{dr}\right) \\ &\quad + \alpha_4 r^2 Z\left(\frac{d^2 f_z}{dr^2}\right) + 2Pqf_p r^2 - 2\alpha_4 Z q^2 f_z r^2 = 0, \end{aligned} \tag{22}$$

$$\begin{aligned} &-U f_u \omega r^4 + R f_r r^4 - \lambda_p U B_0\left(\frac{df_u}{dr}\right) r^3 + \lambda_p U r^4 K q^4 f_u \\ &\quad - \lambda_p U r^2 K q^2 f_u - \lambda_p U r^4 B_0\left(\frac{d^2 f_u}{dr^2}\right) = 0, \end{aligned} \tag{23}$$

where f_r, f_z, f_u, and f_p are all dependent solely on the radial distance r. We cannot readily see an analytical solution for these equations, yet a sixth order ordinary differential equation for $f_u(r)$ can be found by eliminating f_r, f_z, and f_p from the above equations. Again, solving this analytically poses some problems but we can find a series solution for $f_u(r)$ of the form

$$f_u(r) = \sum_{i=0}^{n} (\mathcal{A}_i r^{-\phi} + \mathcal{B}_i r^{\phi} + \mathcal{C}_i \ln r + \mathcal{D}_i) r^i, \tag{24}$$

where $\phi = q\sqrt{-K_1/B_0}$ and $\mathcal{A}_i$, $\mathcal{B}_i$, $\mathcal{C}_i$, and $\mathcal{D}_i$ are lengthy expressions involving the physical parameters. At this stage, we have taken a series of order 15 in r; at later stages we may neglect order to give the general idea of a particular argument. Since K_1, B_0 and q are positive (assuming a stable configuration) then ϕ must be a complex number and consequently r^{ϕ} is also complex for $r \neq 0$. Since we require v_u to be real, we must set $\mathcal{A}_i = \mathcal{B}_i = 0$. We note here than $f_u(0) = 0$, i.e., there are no undulations of the layers at the centre of the cylinders. Inserting the solution (24) into (23) allows us to compute $f_r(r)$. We can then calculate $f_z(r)$ from (20) and in turn calculate $f_p(r)$ from (22). Also, adding the extra requirement that the system is consistent for all values of r then forces $\mathcal{C}_i$ to be set to zero, thereby arriving at a full solution to our problem. At $r = 0$ we see that there is no radial velocity, yet some azimuthal velocity. Understanding that there should be no pressure changes at $r = 0$ allows us to combine any remaining arbitrary constants into one single constant. Inserting our four solutions into the linear momentum equation in the radial direction (21) gives us an equation in terms of ω and the spatial wave number q. At this stage we reduce the order of the series solution to illustrate the behaviour of ω. To third order in r, the defining equation for ω from (21) is given by

$$a\omega^2 + b\omega + c = 0, \tag{25}$$

where

$$a = 48K_1\rho\lambda_p\alpha_4 q^2 + 4q^4K_1^2\rho^2\lambda_p^2 + 192\alpha_4^2 + 136K\,\rho^2\lambda_p^2B_0q^2 + 432\alpha_4B_0\rho\lambda_p + 900\rho^2\lambda_p^2B_0^2, \tag{26}$$

$$b = -(568q^4K_1\alpha_4\rho\lambda_p^2B_0 + 408\alpha_4^2\lambda_pKq^4 + 900\alpha_4\rho\lambda_p^2B_0^2q^2 + 216\alpha_4^2B_0\lambda_pq^2 + 52q^6K^2\alpha_4\rho\lambda_p^2), \tag{27}$$

$$c = 250q^6K_1\alpha_4^2\lambda_p^2B_0 - 68q^4K_1B_0\alpha_4\lambda_p + 217q^8K^2\alpha_4^2\lambda_p^2 - 450B_0^2\alpha_4\lambda_pq^2 + 225q^4\alpha_4^2\lambda_p^2B_0^2 - 2q^6K_1^2\alpha_4\lambda_p. \tag{28}$$

For stability we require Re(ω) to be positive and we therefore require the quadratic discriminant in ω of (25) to be positive. It is easy to see that $a > 0$ and $b < 0$, hence it can be seen that we require $c > 0$ for stability. Considering the material parameter values

$$B_0 = 10^6\,\mathrm{N\,m^{-2}}, \quad \alpha_4 = 5\times10^{-2}\,\mathrm{Pa\,s}, \quad \rho = 10^3\,\mathrm{kg\,m^{-3}}, \quad K = 5\times10^{-2}\,\mathrm{N} \quad \text{and} \quad \lambda_p = 10^{-15}\,\mathrm{N^{-1}\,m^4s^{-1}}, \tag{29}$$

we see that the coefficient of q^4 and q^6in the expression given by (28) are positive. Hence we are left with the sufficient inequality

$$217q^8K^2\alpha_4^2\lambda^2 - 450B_0^2\alpha_4\lambda q^2 > 0. \tag{30}$$

Therefore, $q \gtrsim 10^8$ is a sufficient restriction on the wave number required for a stable solution of the system, which we can see (to 8th order in r) in Figure 2. This result compares favourably with the result by Kléman & Parodi [3] in which they state, through an elementary energy analysis, that only values of $q \approx 10^7$ would give a stable decaying perturbation. Our analysis shows that $q \gtrsim 10^8$ is a sufficient lower bound for stable perturbations. Kléman and Parodi suggest that this

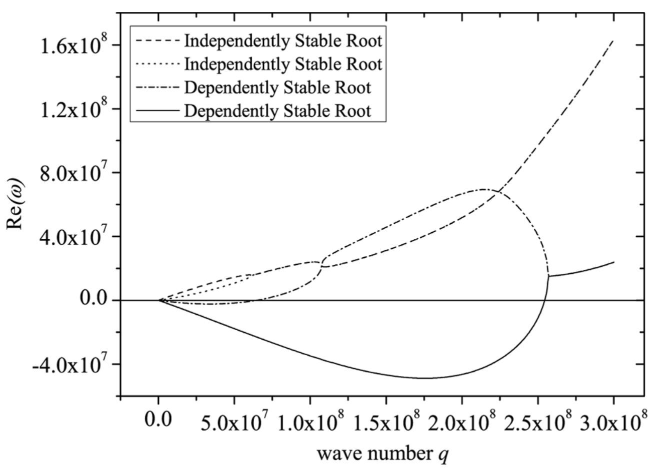


FIGURE 2 The dependence of Re(ω) upon q to 8th order in r. We see here a typical value for q which is required for all possible roots ω to be positive for the parameters stated previously in (29).

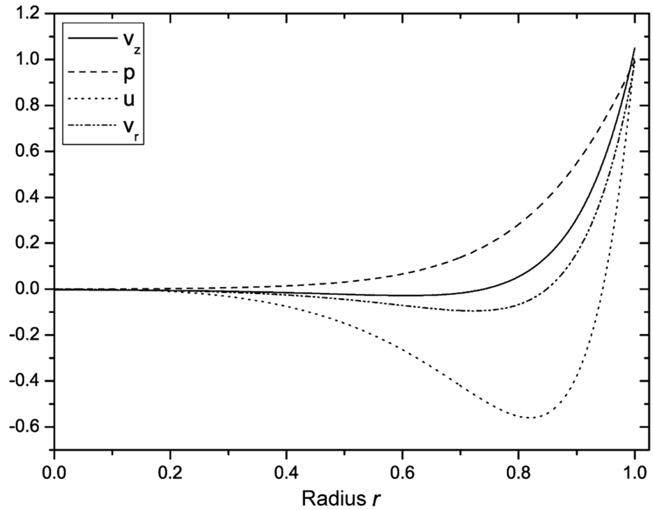


FIGURE 3 Plots of the series solutions for each of the functions discussed in the text, based upon the series solution for the layer undulation displacement u shown in (24). Plots are normalised with respect to sample depth and are calculated with $q = 2.5 \times 10^8$, which gives stability at $t = 0$. Note that due to the nature of series solutions, the growth at large r can be disregarded. These spatial profiles will decay to zero with a relaxation time of $\tau_r = 1/\omega \approx 70\,\mu s$ for the material parameters stated in (29).

value of q is so large that it is not physically realisable. However, we note that this gives a response time $\tau_r \approx 70\,\mu s$, which is strikingly similar to the results commented on by Stewart [13] for a SmC* sample in which the response time is calculated as $\tau_r \approx 50\,\mu s$. Plots of the solutions for the four variables are found via Figure 3.

4. CONCLUSIONS AND FURTHER WORK

Using the dynamic equations proposed in [7] we have derived a set of governing equations for a cylindrically layered sample of SmA. Using these equations, and employing a suitable perturbation ansatz, we derived expressions for the velocity, pressure and layer undulations in the radial velocity only case. However, no information on the decay rate ω was found. This motivated us to look at functions involving r and z, in which we can prove stability with a lower bound in the spatial wave number q, confirming the previous elementary analysis by Kleman and Parodi [3].

In this work we have assumed the layer normal **a** and the director **n** always coincide, i.e., $\mathbf{a} \equiv \mathbf{n}$. However, we shall extend our work so that

a and **n** may be decoupled in the bending of the smectic layers. This will be achieved by a further application of the dynamic equations proposed by Stewart [7]. Early results suggest that stability will be ensured in a similar manner [14]. Also, with a view to developing this work further, we shall expand the viscous stress tensor to

$$\tilde{t}_{ij} = \alpha_4 A_{ij} + \tau_1 (a_k A_{kp} a_p) a_i a_j + \tau_2 (a_i A_{jp} a_p + a_j A_{ip} a_p), \tag{31}$$

where τ_1 and τ_2 are SmA-like viscosities and also consider the full viscous stress tensor proposed by Stewart [7]. Preliminary indications are that stability will generally occur for restricted ranges of viscosities that obey a priori restrictions derived from a dissipation inequality.

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